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Constitutive modeling of monotonic behavior of clays: Mathematical formulation, numerical implementation and experimental verification

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***Yazar (Author)**: Mehmet Barış Can ÜLKER*

ORCID: 0000-0001-7632-2303

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Killerin Statik Bünye Davranışlarının Modellenmesi: Matematiksel Formülasyon, Sayısal Uygulama ve Deneysel Doğrulama

Araştırma Makalesi / Research Article

Mehmet Barış Can ÜLKER*
Deprem Mühendisliği ve Afet Yönetimi Enstitüsü, Deprem Mühendisliği Bölümü, İstanbul Teknik Üniversitesi, Türkiye

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ÖZ

İnşaat mühendisliği sistemlerinin tasarımında temeller önemli bir yer tutar. Dış yüklemeler altında yüzeysel ve derin temellerin stabilite kaybına neden olacak durumların belirlenmesinde geoteknik incelemeler ayrıca önemlidir. Temel tabakasının kil olması durumunda, yükleme sırasında göçmeye neden olan koşullar tanımlanmalıdır. Bunu yapmanın en doğru yolu, matematiksel denklemler yardımıyla zeminin bünye davranışını teorikleştirmektir. Bu çalışmada, statik yükler altında kil zeminin bünyesel davranışı Genelleştirilmiş Plastisite Teorisi ile modellenmiştir. Sayısal formülasyon, her bir yükleme aşaması için açık integrasyon yöntemi ile çözülen temel denklemler cinsinden özetlenmiştir. Çözümün yapılabilmesi için bir bilgisayar programı geliştirilmiştir. Zeminde elasto-plastik matris, şekil değiştirme-gerilme ilişkisinin tersi alınarak türetilmiş, bu sayede kile ait gerilme-şekil değiştirme ilişkisi modelde herhangi bir akma veya potansiyel fonksiyon kullanmadan artımsal olarak elde edilmiştir. Sonrasında zeminde kalıcı şekil değiştirmeler hesaplanmıştır. Ardından, modeli ve bilgisayarda uygulamasını doğrulamak için bir dizi drenajlı ve drenajsız üç eksenli deformasyon kontrollü deney simüle edilmiştir. Deneyler, Genelleştirilmiş Plastisite modelinin kapasitesini belirlemek amacıyla, iyi bilinen modifiye-Cam-kili modeli ile de simüle edilmiştir. Simülasyon sonuçları, modelin normal ve aşırı konsolide killerin statik davranışlarını yakalamadaki etkinliğini ve kapasitesini göstermektedir.

Anahtar Kelimeler: Killer, bünyesel modelleme, genelleştirilmiş plastisite, statik yükleme, sayısal analiz.

Constitutive Modeling of Monotonic Behavior of Clays: Mathematical Formulation, Numerical Implementation and Experimental Verification

ABSTRACT

Foundations constitute a significant part of the design of civil engineering systems. Geotechnical considerations are particularly important in identifying the conditions leading to instability of shallow and deep foundations under various loadings. In the case the foundation layer is clay, one should identify the conditions leading to failure of clay soil upon loading. The most common way of doing so is to theorize the constitutive behavior of the soil using mathematical equations. In this study, constitutive modeling of clays under monotonic loadings is presented using the Generalized Plasticity Theory. Numerical formulation is summarized in terms of governing equations which are solved for each load step by an explicit integration method which is implemented into a computer program. Elasto-plastic constitutive matrix is derived based upon the inversion of strain-stress relationship without using a yield or a potential function in the model which is used to get the stress-strain incremental relationship. Plastic strains are then calculated using a non-associative flow rule. Subsequently, a number of drained and undrained strain-controlled triaxial tests are simulated to verify the model and its implementation. The related tests are also simulated using the well-known modified Cam Clay model to highlight the capabilities of the Generalized Plasticity model. Simulation results demonstrate the effectiveness and the capability of the model to capture static behavior of normally and overconsolidated clays.

Keywords: Clays, constitutive modeling, generalized plasticity, monotonic loading, numerical analysis.

1. INTRODUCTION

Stress-strain relationship and strength properties of soils under static loads must be known in the solution of geotechnical engineering problems. The shear behavior of natural soils under applied loads is highly dependent on the type of soil, drainage conditions and effective mean stress. In addition, the change in point-to-point relations of soil layers or also called heterogeneity and the change in engineering properties of soil in different directions (anisotropy) necessitate laboratory tests to be carried out. This allows us to determine the shear strength properties of the soil as well as the stress-strain relationship. However, it is not easy to conduct a different experiment each time one needs to determine the engineering properties of a soil sample prior to the solution of a geotechnical engineering problem. While it is still necessary to do so, engineers find themselves in a tough spot requiring it to make yet some kind of

**Sorumlu Yazar (Corresponding Author)*
e-posta : mbulker@itu.edu.tr

generalization in representing the behavior of soils under field load conditions. In this respect, firstly, the soil behavior should be modeled theoretically starting from the response under monotonic loading. Such attempts resulted in the development of various constitutive models over the last few decades. From these models, Generalized Plasticity Theory (GPT) can be considered to have the least possible complexity with most possible accuracy [1-5]. That is, the model makes predictions that require the least number of parameters to satisfy an acceptable level of accuracy. GPT not only predicts the necessary information of soil behavior sufficiently, but it does this in a simple enough manner. That is, the model presents plausible and repeatable arguments. For example, the Classical Plasticity and the Bounding Surface Plasticity, which are both the special cases of the GPT, require still many model parameters and mathematical functions to be defined. Yield and plastic potential functions are two common examples of such required mathematical relations currently used in many classical models.

One other advantage of using the GPT framework is that, it allows development of new models with different functions. This way, given the need one can always employ such functions in GPT for modeling a particular stress state or a soil test. This feature of the model makes it flexible enough to model the nonlinear static and dynamic behavior of both cohesionless and cohesive soils. Modifications and expansions made to the theory in recent years, made it more appealing to use in the numerical solution of some key problems encountered in geotechnical engineering [6].

In this study, constitutive modeling of clays under monotonic loadings is presented using the Generalized Plasticity Theory. Numerical formulation is summarized in terms of governing set of constitutive equations which are solved for each load step by an explicit integration method. General equations of the the theory written in terms of stress-strain relationship, flow rule and the hardening law are presented. Plastic strains are calculated using a non-associative flow rule without referencing a yield or a potential function. Elastoplastic constitutive matrix is derived by the inversion of strain-stress relationship without using the consistency condition. Subsequently, a number of drained and undrained strain-controlled triaxial tests are simulated to verify the model and its implementation.

2. GOVERNING CONSTITUTIVE EQUATIONS

Generalized Plasticity Model (GPM) is the constitutive model formulation of the GPT framework. In this section, the main features of the model is described in terms of how the loading direction is decided, unit vector definitions are made, stress-strain relationship is derived and flow rule and the hardening law are defined.

2.1 Loading Direction

The loading and unloading steps of the model is decided in terms of associated unit vectors. The way the unit vectors are defined is that, they are prescribed to be normal to a presumable surface that does not have to exist in the stress space which allows to draw two important conclusions. One is the clear distinction in the loading directions through:

$$d\sigma_{kl}' : n > 0 \rightarrow loading \tag{1a}$$

$$d\sigma_{kl}' : n < 0 \rightarrow unloading \tag{1b}$$

$$d\sigma_{kl}' : n = 0 \rightarrow neutral\ loading \tag{1c}$$

where $d\sigma_{ij}'$ is the change in effective stress and n is the unit vector along the direction of stress increment. These relations are valid only for hardening materials and should be changed [5] in the case of softening with the introduction of $d\sigma_{ij}^e$ calculated using the elastic strains. It should be noted here that throughout this paper, an effective stress notation of stresses will be followed.

The other important feature of using unit vectors is that, the entirety of the theory is now dependent highly on these unit vectors and it is indeed possible to construct a constitutive model with the arguments developed in this manner by making use of their definitions. For instance, we have the luxury of being able to write the necessary relations for both loading (L) and unloading (U) cases with the help of a continuity condition which is now solely a mathematical construct as opposed to having a physical meaning in the case of a yield surface. The strain-stress relationship is the key to start off the basic formulation of the model. We write:

$$d\varepsilon_{ij} = C_{ijkl}^{L} d\sigma_{kl}' \tag{2a}$$

$$d\varepsilon_{ij} = C_{ijkl}^{U} d\sigma_{kl}' \tag{2b}$$

Continuity between loading and unloading requires that constitutive tensors C_{ijkl}^{U} and C_{ijkl}^{U} are of the form:

$$C_{ijkl}^{L} = C_{ijkl}^{e} + \frac{1}{H_L} n_g^L \otimes n \tag{3a}$$

$$C_{ijkl}^{U} = C_{ijkl}^{e} + \frac{1}{H_U} n_g^U \otimes n \tag{3b}$$

where C_{ijkl}^{e} is the elastic compliance matrix and $n_g^{L/U}$ is the unit vector showing the direction of plastic straining. It can be readily shown that in neutral loading, both relations of (3) are equal and hence non-unique definitions of strain is avoided. Therefore we write:

$$d\varepsilon_{ij}^{L} = C_{ijkl}^{e} d\sigma_{kl}' \tag{4a}$$

$$d\varepsilon_{ij}^{U} = C_{ijkl}^{e} d\sigma_{kl}' \tag{4b}$$

2.2 Unit Vectors and Stress Dilatancy

As mentioned, the GPM relies on the way two unit vectors, $n_g^{L/U}$ and n, are defined. While $n_g^{L/U}$, n show the direction of plastic flow and stress increment, respectively. Depending upon whether they are chosen as equal (associated model) or not (non-associated model), they are defined as:

$$n_v = \frac{d}{\sqrt{1+d^2}}, \quad n_s = \frac{1}{\sqrt{1+d^2}}, \quad n = (n_v, n_s) \tag{5a}$$

$$n_{gv} = \frac{d_g}{\sqrt{1+d_g^{\,2}}}, n_{gs} = \frac{1}{\sqrt{1+d_g^{\,2}}}, \quad n_g = (n_{gv}, n_{gs}) \tag{5b}$$

where subscripts ‘v’ and ‘s’ stand for volumetric and shear, respectively. In the above, d and d_g are dilation ratios that are functions of slopes of state lines, M and M_g, but more definitely defined as $d=d\varepsilon_v^p/d\varepsilon_s^p$ where $d\varepsilon_v^p$ is the volumetric plastic strain increment and the $d\varepsilon_s^p$ is the deviatoric plastic strain increment. As a result of tests on Bangkok clay by [7] under constant stress ratio, $\eta=q/p'$, a linear relationship (see Figure 1) is found for d as,

$$d = (1+\mu)(M-\eta) \tag{6}$$

where μ is a constant to be determined from the best line fit of experimental data of η vs. d and M is the slope of the critical state line. In the case of non-associated plasticity, this relation becomes:

$$d_g = (1+\mu)(M_g-\eta) \tag{7}$$

where M_g is the slope of the line for which there is no volumetric expansion (Figure 2). M and M_g are dependent on the Lode’s angle, θ [8], and signifies that the GPM is in harmony with the *critical state soil mechanics*. We know that the residual state is also controlled by M_g in soils. Sign of the slopes in (6) and (7) may change depending upon the loading direction.

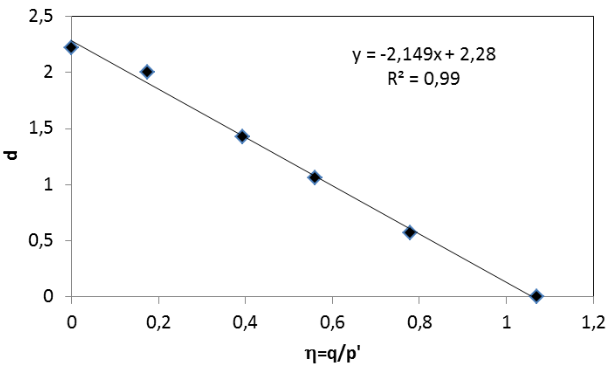

Figure 1. Dilatancy-stress ratio relationship, d-η, for Bangkok clay, (regenerated from [7])

2.3 Flow Rule

Plastic strain increments are calculated through the flow rule considering the classical plasticity. First, decomposition of strains is written as:

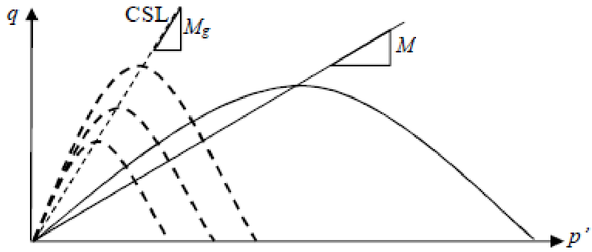

Figure 2. Yield and potential functions in soils

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{\ e} + d\varepsilon_{ij}^{\ p} \tag{8}$$

where the elastic and plastic strains are calculated as:

$$d\varepsilon_{ij}^{\ e} = C_{ijkl}^{e} d\sigma_{kl}' \tag{9a}$$

$$d\varepsilon_{ij}^{\ p} = \frac{1}{H_{L/U}}\left[n_g^{L/U} \otimes n\right] : d\sigma_{ij}' \tag{9b}$$

where (9a) is similar to (2) and in (9b) $H_{L/U}$ is the plastic modulus. Again we notice the use of two unit vectors in defining the plastic strain.

2.4 Stress-Strain Relationship

Effective stress increments, $d\sigma_{ij}'$, are calculated through the stress-strain relationship as:

$$d\sigma_{ij}' = D_{ijkl} d\varepsilon_{kl} \tag{10}$$

where D_{ijkl} is the fourth order elasto-plastic material matrix (also called constitutive matrix) and $d\varepsilon_{kl}$ is the total strain increment. Since there is no yield surface prescribed in the model, there is no consistency condition written to enforce the stress vector to be on a yield or a similar form of surface. Owing to the need to write explicitly Eq. (10), we first rewrite (8) using (9) and (10) as:

$$d\varepsilon_{ij} = \left\{C_{ijkl}^{e} + \frac{1}{H_{L/U}}\left[n_g^{L/U} \otimes n\right]\right\} : d\sigma_{kl}' \tag{11}$$

where the term in the parenthesis is the elastoplastic compliance tensor, C_{ijkl}^{ep}. Now, inverting Eq. (11) with some vector algebra we get,

$$d\varepsilon = C^e d\sigma' + \frac{n_g^{L/U}\left(n^T D^e d\varepsilon\right)}{H + n^T D^e n_g^{L/U}} \tag{12}$$

which results in the following final stress-strain relationship for both loading and unloading stages:

$$d\sigma' = \left[D^e - \frac{D^e n_g^{L/U} n^T D^e}{H_{L/U} + n^T D^e n_g^{L/U}}\right] d\varepsilon_{L/U} \tag{13}$$

Although this relation is no different than the one used in various sources derived from the classical plasticity, the fact that no yield or any other surface (say F=0 type) is required to derive it makes the GPT a powerful, yet a simple theory. While it is not required, GPT also allows such a F=0 surface to be implemented to calculate n [9-

10]. In the case of an undrained loading, pore pressure increments (dp_w) are computed in terms of the mean and deviatoric stress components ($d\sigma_v', d\sigma_s'$) and confining stress $d\sigma_c'$ as,

$$dp_w = d\sigma_s'/3 + d\sigma_c' - d\sigma_v' \tag{14}$$

2.5 Hardening Law

2.5.1 Normally Consolidated Clays

In order for the evolution of plastic strains, hence the stress-strain relationship, hardening law must be defined. The ongoing model is of *isotropic hardening* one with a deviatoric plastic strain increment in its nature which can also be called a *deviatoric hardening model*. The plastic modulus for isotropic virgin compression paths is,

$$H_L = H_0 \sigma_v' \tag{15}$$

where $\sigma_v' = I_1'$ and $H_0 = \dfrac{1+e_0}{\lambda - \kappa}$ with e_0 being the initial void ratio, λ is the slope of virgin compression curve and κ is the slope of unloading/reloading curve. If one is to follow other stress paths, this relation becomes:

$$H_L = H_0 H_\eta \sigma_v' \tag{16}$$

where

$$\begin{aligned} \eta = 0 &\longrightarrow H_\eta = 1 \\ \eta = M &\longrightarrow H_\eta = 0 \end{aligned} \tag{17}$$

A more general stress path function is provided as below to ensure that similar stress paths yield similar results,

$$H_\eta = \left(1 - \frac{\eta}{M}\right)^\alpha \frac{\left(1+d_0^{\,2}\right)}{\left(1+d^2\right)} \left| sign\left(1 - \frac{\eta}{M}\right)\right| \tag{18}$$

with α=2 for most clay soils. In order for this plastic modulus definition to be three dimensional (3-D) so it can be used in an all purpose finite element code, M should be a function of Lode's angle such as the one below that is suitable also for a smoothed Mohr-Coulomb model;

$$M = \frac{18M_c}{18 + 3\left(1 - \sin 3\theta\right)} \tag{19}$$

M_c is the value of M in compression.

2.5.2 Over-Consolidated Clays

This much of the model is sufficient to model normally consolidated clays. For the over-consolidated soils (OC), there has to be some kind of a *memory parameter* to keep track of previous stress history. In this study, this is achieved by a *mobilized hardening modulus*, H_ζ defined as:

$$H_\zeta = \left(\frac{\zeta_{\max}}{\zeta}\right)^\gamma \tag{20}$$

where

$$\zeta = \left(1 - \frac{(1+\mu)}{\mu}\frac{\eta}{M}\right)^{-1/\mu} \frac{\eta}{M}\sigma_v' \tag{21}$$

is the *mobilized stress function*. The plastic loading modulus, H_L, is now modified as:

$$H_L = H_0 H_\zeta \left(H_\eta + H_\xi\right)\sigma_v' \tag{22}$$

where other hardening parametes are:

$$H_\xi = \beta \exp\left(-\beta\xi\right) \tag{23}$$

where

$$\beta = \beta_0 \left(1 - \frac{\zeta}{\zeta_{\max}}\right) \tag{24}$$

and

$$d\xi = \sqrt{\left(d\varepsilon_s^p : d\varepsilon_s^p\right)} \tag{25}$$

We see that two additional fitting parameters, γ and β_0, are needed to extend the range of application of the model, particularly to modeling OC clays.

2.6 Elastic Behavior

Since soils are effective normal stress dependent materials where their stiffness moduli changes as the depth of the soil layer increases. Therefore, their elastic moduli, namely the shear modulus, G and the bulk modulus, K, are taken as a function of mean effective stress, p'. The following elastic relations hold:

$$d\varepsilon_s^e = \frac{1}{G} d\sigma_s \tag{26a}$$

$$d\varepsilon_v^e = \frac{1}{K} d\sigma_v \tag{26b}$$

where

$$G = G_0 \left(\frac{p'}{p_0}\right) \tag{27a}$$

$$K = K_0 \left(\frac{p'}{p_0}\right) \tag{27b}$$

with p_0 being the reference pressure, G_0 the initial shear modulus and $K_0 = \dfrac{1+e_0}{\kappa}$ the initial bulk modulus. A recent comprehensive study investigating the effect of nonlinear elastic behavior on the elasto-plastic response of soils is [11].

5. MODELING TRIAXIAL TESTS

In this study, the main framework of the GPM is implemented in a computer program written in MATLAB which is verified with available monotonic triaxial shear tests. In a standard triaxial compression test, Lode's angle, θ takes the following form:

$$-\frac{\pi}{6} < \theta = \frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}J_3'}{2J_2'^{3/2}}\right) < \frac{\pi}{6} \tag{28}$$

where $J_2' = \frac{1}{2}\sigma_s' : \sigma_s'$ and $J_3' = \frac{1}{3}tr\left(\sigma_s'^3\right)$ with $\sigma_s' = \mathrm{dev}(\sigma') = \sigma' - \frac{1}{3}tr(\sigma')$. In triaxial stress space, we use p' and q as stress variables with $\sigma_v' = I_1' = p'$ and $q = \sqrt{3J_2'}$. As for strains, we write,

$$d\varepsilon_v = tr(d\varepsilon) \tag{29}$$

and

$$d\varepsilon_s = \frac{2}{3}\sqrt{\left[\frac{1}{2}\mathrm{dev}(d\varepsilon) : \mathrm{dev}(d\varepsilon)\right]} \tag{30}$$

where

$$\mathrm{dev}(d\varepsilon) = d\varepsilon - \frac{1}{3}d\varepsilon_v \tag{31}$$

3.1 Normally Consolidated Behavior

Prior to describing the modeling characteristics of the theory upon triaxial shear response of normally consolidated clays, it should be noted that the current formulation of the GPM is also capable of capturing the isotropic or anisotropic compression of clay soils. Such a response is modeled while the sample is still on normally consolidated line, NCL and recompression line during loading-unloading stages. An example simulation is run by [2] using a Kaolin soil with ϕ'=23^0. Slightly smaller void ratio changes are observed in their analyses as compared to the tests of [12].

There are two types of triaxial tests simulated in this section. One is the constant p' test and the other is the constant cell pressure, σ_c' test. Both of them are performed drained and undrained. In the constant p' test, following relation holds for the principal stress increments:

$$d\boldsymbol{\sigma}_2' = d\boldsymbol{\sigma}_3' = -d\boldsymbol{\sigma}_1'/2 \tag{32}$$

In the constant σ_c' test (or constant σ_3' test) we have the following constraint conditions for the stress and strain components:

$$d\boldsymbol{\sigma}_2 = d\boldsymbol{\sigma}_3 = 0;\ d\boldsymbol{\varepsilon}_2 = d\boldsymbol{\varepsilon}_3 = -d\boldsymbol{\varepsilon}_1/2 \tag{33}$$

These constraints must hold true throughout the analysis to be able to get accurate results. Figure 3 presents the consolidated drained (CD) triaxial test simulation done by keeping constant p' of a normally consolidated (NC) Bangkok clay in terms of stress ratio-deviatoric strain as well as stress ratio-volumetric strain behaviors. Test results of [7] are shown in markers. The model simulates the stress-strain relationship well enough (Figure 3a) but underpredicts the volumetric strain behavior (Figure 3b).

Due to this reason, the normally consolidated behavior is also modeled using the modified Cam-Clay model (MCC) to better appreciate the differences between the two related but inherently different models. The MCC assumes that in addition to reaching the critical state for stresses, soil material needs also to be at a loose state to consider failure. Thus, at failure we have:

$$q_f = M\,p_f' \tag{34}$$

$$v_f = \Gamma - \lambda\ln\left({p'_f}/{p_{ref}}\right) \tag{35}$$

where (p_f', q_f) are the stresses at failure and p_{ref} is the reference pressure, M is the slope of critical state line, Γ, is the specific volume at the reference pressure and v_f is the specific volume at critical state. In the MCC model the isotropic hardening is considered where the compression behavior of soil is governed by the following relationships in loading and unloading, respectively:

$$v = N - \lambda\ln({p'}/{p_{ref}}) \tag{36}$$

$$v = v_s - \kappa\ln({p'}/{p_c}) \tag{37}$$

where N is the specific volume at 1 atm pressure and v_s is the specific volume at the beginning of unloading. p_c is the preconsolidation pressure.

Figure 4 shows the results of constant cell pressure, σ_c', consolidated undrained (CU) triaxial test simulation for the same NC clay. GPM results capture the deviatoric stress-strain behavior as well as the stress path very well. While the pore pressure response yields some discrepancy, the maximum pore pressure is captured by the GPM. In comparison, the MCC model predicts the pore pressures slightly better but the stress path slightly worse.

The drained test results are given in Figure 5 in terms of stress ratio-deviatoric strain behavior and stress ratio-volumetric strain relationship. CD results match better than the CU test results as it was easier to model the drained behavior of the NC Bangkok clay. As for the MCC model, volumetric strains are modeled better than they are by the GPM.

Figure 6 shows the constant cell pressure CU test [13] simulations of NC Weald clay. While the failure load is slightly underpredicted by the model, pore pressure is overpredicted, in turn. As far as the NC clay behavior, Figure 7 is the last test simulation with a constant σ_c' CD test. The simulation results capture the overall behavior very well with a remarkable match of the volumetric-strain vs. axial strain relationship (Figure 7b). Table 1 gives the model parameters used in the analyses including the overconsolidated (OC) ones.

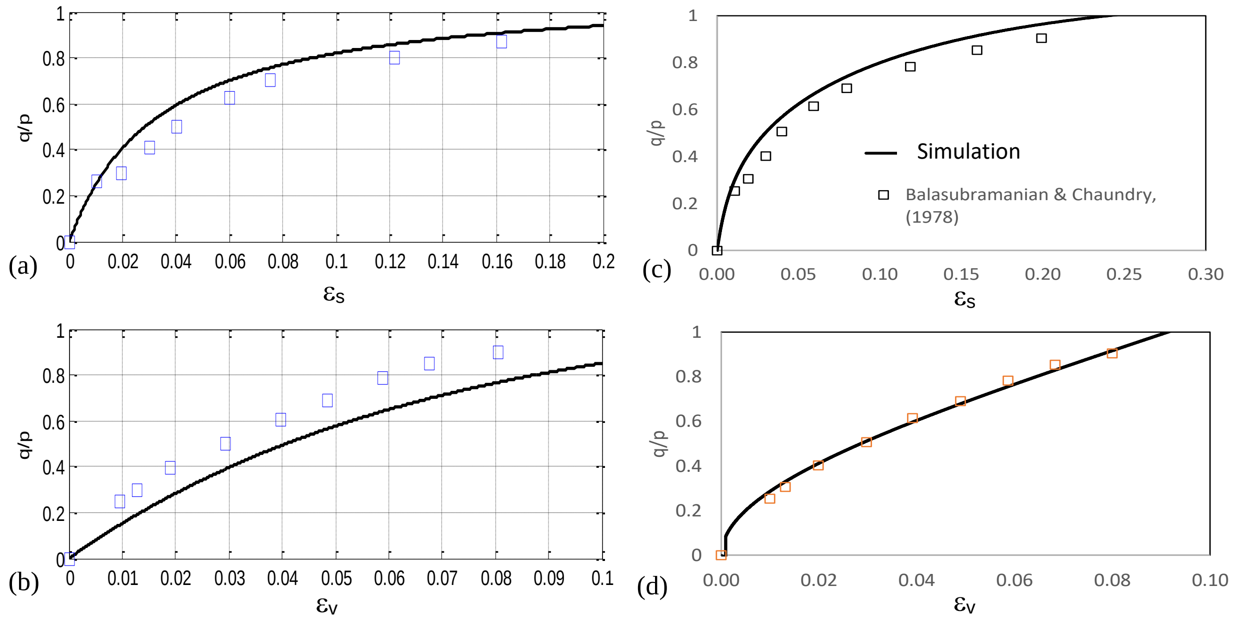

Figure 3. Constant *p*' CD simulation for NC Bangkok clay; stress ratio-deviatoric strain behavior and stress ratio-volumetric strain behavior; (a) and (b) GPM predictions, (c) and (d) MCC predictions

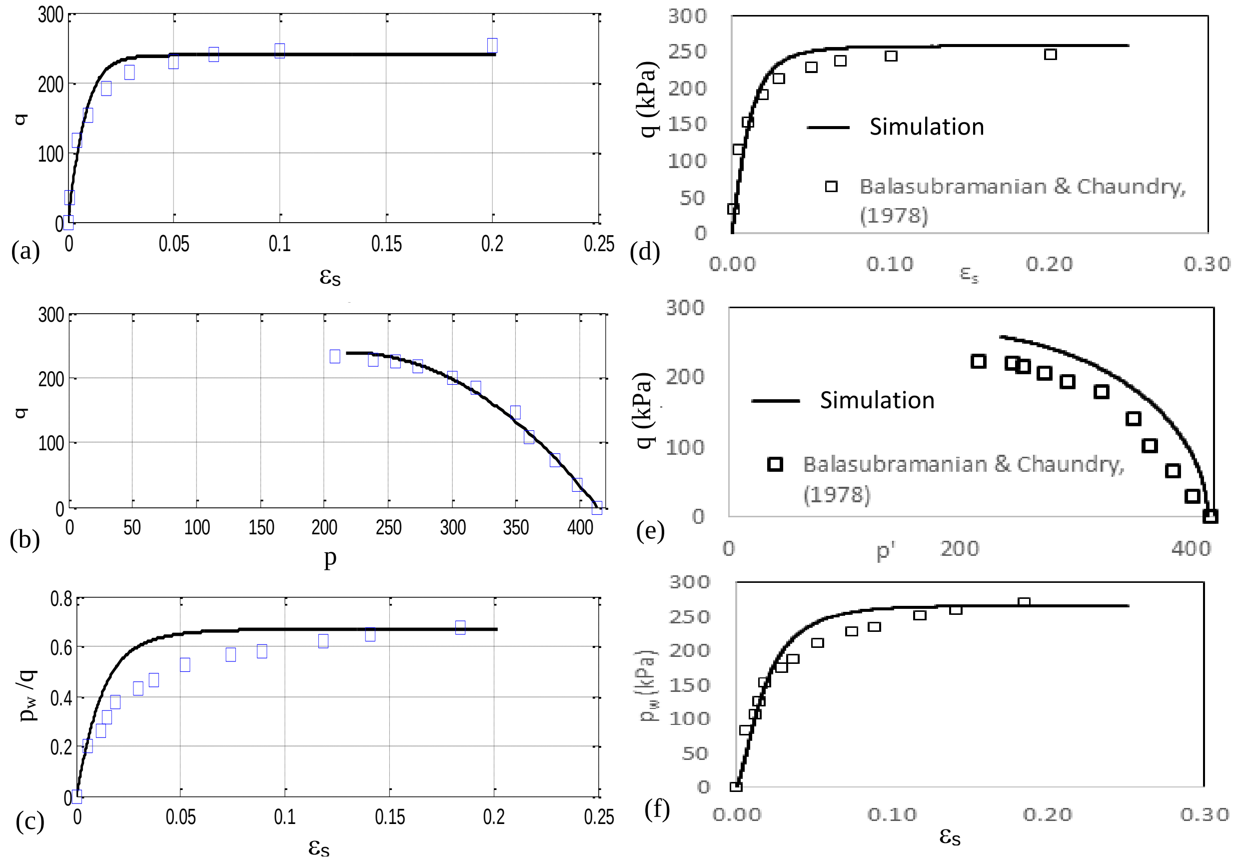

Figure 4. Constant σ_c' CU simulation for NC Bangkok clay; stress ratio-deviatoric strain behavior and stress path and pore pressure-deviatoric strain behavior; (a)-(c) GPM predictions, (d)-(f) MCC predictions

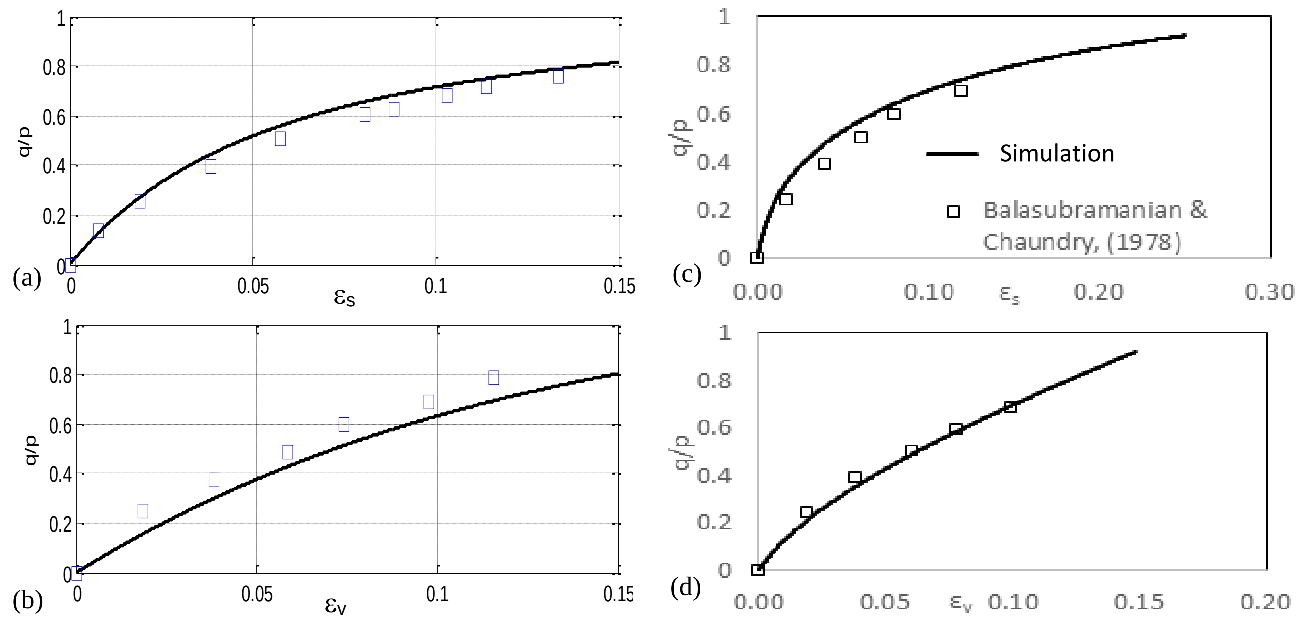

Figure 5. Constant σ_c' drained test (CD) simulation for NC Bangkok clay, Stress ratio-deviatoric strain behavior, stress ratio-volumetric strain behavior; (a)-(b) GPM predictions, (c)-(d) MCC predictions

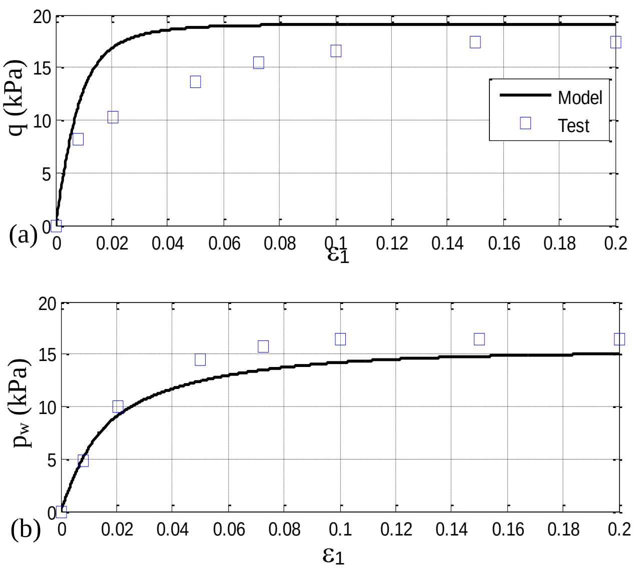

Figure 6. Constant σ_c' undrained test (CU) simulation for NC Weald clay, a) Stress-axial strain behavior, b) Pore pressure-axial strain behavior, [13] tests in markers

3.2 Over-Consolidated Behavior

With the additional 'memory' parameter in the evaluation of the plastic hardening modulus, the current model is capable of capturing some basic properties of OC clays also, even for a very high overconsolidation ratio (OCR). Figure 8 and 9 present such a highly OC Weald clay triaxial test results with OCR=24. Tests modeled are again the constant σ_c' undrained and drained tests. The GPM is able to simulate the sign changes in the pore pressure and volumetric strain behavior (Figure 8b, 9b) and capture the overall stress-strain response (Figure 8a, 9a). The only drawback of the OC clay behavior is that there is a residual response beyond the peak point with a lower stress magnitude which is hard to model with the current formulation (Figure 9a). Surely, one can add more parameters into the model that will only take place once the peak is reached however, that will further complicate the model and therefore is avoided in this study.

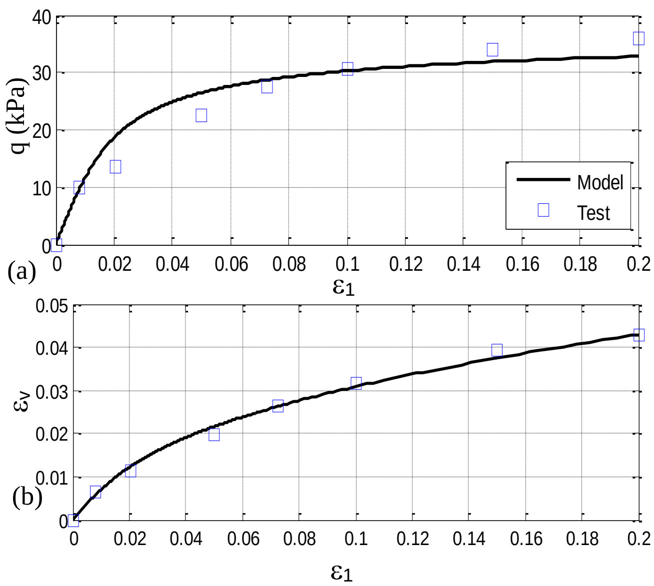

Figure 7. Constant σ_c' drained test (CD) simulation for NC Weald clay, a) Stress-axial strain behavior, b) Volumetric strain-axial strain behavior, tests of [13] in markers

Table 1. Model parameters used in triaxial test simulations

Soil Type	K_0 (kPa)	G_0 (kPa)	p_0 (kPa)	M	H_0	μ	β_0	γ	α
Bangkok Clay	12420	15000	414	1.1	6.6	2.0	-	-	0.75-0.95
Weald Clay	5281	5516	414	0.9	165	3.0	0.1	0.4	0.009-0.15

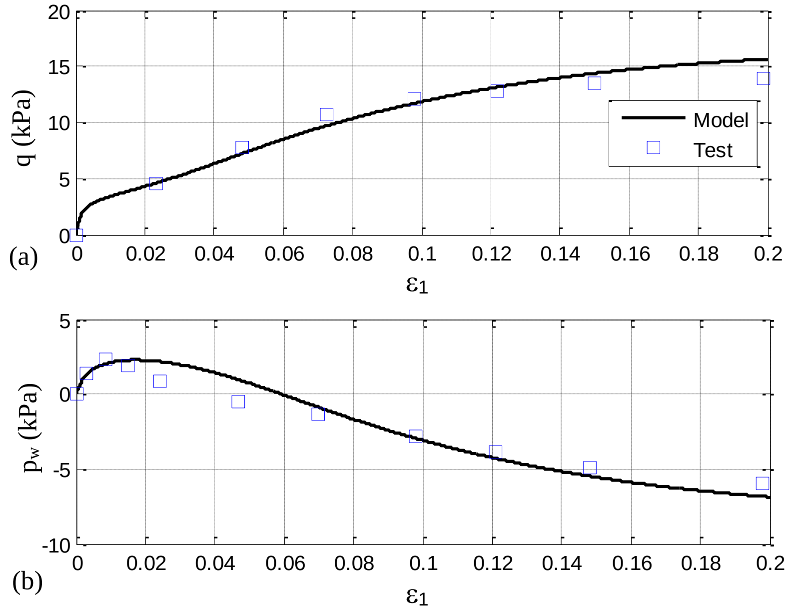

Figure 8. Constant σ_c' undrained test (CU) simulation for OC Weald clay (OCR=24), a) Stress-axial strain behavior, b) Pore pressure-axial strain behavior, tests of [13] in markers

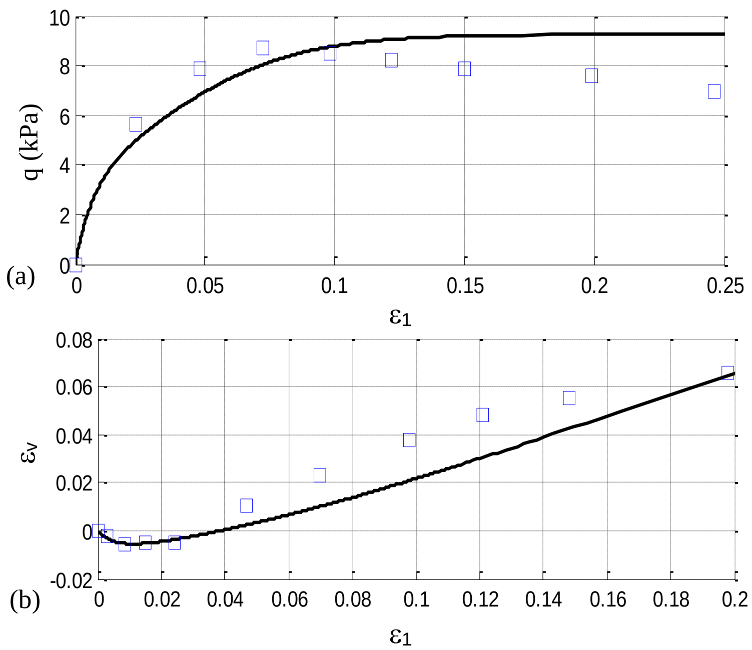

Figure 9. Constant σ_c' undrained test (CU) simulation for OC Weald clay (OCR=24), a) Stress-axial strain behavior, b) Pore pressure-axial strain behavior, tests of [13] in markers

4. CONCLUSION

Constitutive modeling of monotonic behavior of clays is presented in this study. Mathematical formulation of the Generalized Plasticity Model is given in all its aspects in terms of the constitutive relations for clays under monotonic loadings which are integrated using an explicit method. No reference to a yield or a potential surface is made in the model and thus, the flow rule and particularly the elasto-plastic constitutive matrix are derived based upon two unit vectors essentially calculated to describe the plastic flow direction as well as the loading direction. Computer implementation is followed by its experimental verification through a number of drained and undrained triaxial shear tests. At this point, the tests are also simulated using the modified Cam Clay model to make a comparison between the capabilities of the GPM and the classical MCC models. Simulation results indicate that the GPM is a simple but very effective model to capture the static behavior of normally and overconsolidated clays for various stress paths.

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