

# Material Features That Determine the Activity Preferences of Mathematics Teachers

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#### Abstract

Researchers working on activity-based mathematics teaching suggest that the materials deeply shape the design and implementations. Despite this effect, empirical knowledge about how teachers approach the materials in their activity selection, and which features of the materials are decisive in their preferences is quite limited. Hence this study aims to designate material features determinant in teachers' mathematical activity preferences. This research was designed as a multiple case study and was conducted with three secondary school mathematics teachers. During the data collection process, six consecutive semi-structured interviews were conducted with the participants. While structuring the interview process, multiple activities were prepared with different materials to serve the same gain. The data were analyzed by thematic analysis method. The analyses yielded five distinct features that accounted for teachers' selection and preferences of mathematical activities with regard to materials: serving to the mathematical gains, functionality, accessibility, being proficient in use and student familiarity. The findings showed that teachers' activity preferences had a complex structure and pointed out that instructional decisions were not only shaped on a pedagogical basis and were not only concerned with students' mathematical development. Based on the evaluations of the teachers, it was concluded that the predictions about the affordances and constraints of the materials were decisive in the activity selection.

Keywords: Mathematical activity, material features, teacher preferences

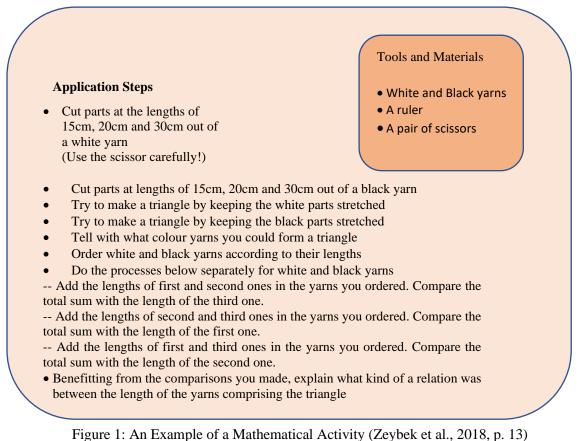
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# Introduction

Activity-based mathematics teaching has been a research focus for many years. For example, since 1980s, studies on the concept of academic tasks coined by Doyle and colleagues (e.g., Doyle, 1983; Doyle & Carter, 1984) have put efforts to determine significant components of activity-based teaching. One reason behind continuous research interest on activity-based mathematics teaching is related to the fact that it encourages students to take the responsibility of their own learning and allows learners to develop rich understandings (Lozano, 2017).

Even though there are varieties in the definitions of mathematical activity (Gürbüz, Pırtıcı & Toprak, 2014; Levenson, Swisa & Tabach, 2018; Margolinas, 2013; Özgen, 2017), there appears an agreement that mathematical activities are structured around an academic task (see Bozkurt et al., 2022; Swan, 2008). An example of an activity that was built on a mathematical task is given in Figure 1. In this activity, colourful yarns were used and by means of instructions, students were asked to undertake certain works. It was aimed in this activity that students to complete an academic task that can be summarized as determining the necessary conditions in order to form a triangle with three-line segments and to discover the concept of triangle inequality.



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A concise definition of mathematical activity comes from Özmantar et al. (2010) who consider mathematical activity as a task in practice. This perspective is shared by Jones and Pepin (2016) who highlight that the process of putting the activities into practice corresponds to a certain pedagogical approach. Such viewed, every activity has a pedagogical potential shaped by the teacher whose preferences determine how this potential is reflected to the students. In this sense, activities are instructional tools that only become meaningful with the practitioners or teachers.

Activity-based teaching, in and of itself, is not a learning model or theory. Rather, it refers to the instructional practices in which activities are included as learning resources (Memiş et al., 2021). Because of its very structure, activity-based teaching aims to hand over responsibility of learning to students. In this respect different approaches and/or theoretical perspectives rely on mathematical activities. For example, structured problems presented as activities found a strong base in the context of

realist mathematics education (Van den Heuvel-Panhuizen & Drijevers, 2020). Similarly, didactical engineering theory (Artigue, 2009) and lesson study (Doig, Groves & Fujii, 2011) place a heavy emphasis on instructional practices structured around activities. In this sense, activities are widely used in teaching practices.

The curricular change that took place in 2005 in Turkey placed a particular importance on mathematical activities as part of classroom practices. Since then, activities and activity-based teaching have constituted an important agenda for especially the secondary education teachers. With this curriculum, activity-based teaching drew attention to a great number of researchers in the field of mathematics education in Turkey and it has become an important research area (Çenberci & Özgen, 2021; Karataş, 2021; Taşpınar-Şener & Bulut, 2022).

One strand of research on activities concerns the features and components of mathematical activities. These studies mostly focus on teaching-learning process and extract effective features of this process in determining components of activities and characteristics of effective activity implementation. As an example, while explaining the determining features in the quality of activities, Smith and Stein (1998) put mathematical development in the centre. These researchers directed their attention on classroom works allowing the development of students mathematical thinking. Margolinas (2013) indicates the importance of practices allowing students to discover the mathematics embedded in the tasks by taking responsibilities of their own learning through active participation in the activities. Powell et al. (2009) evaluate the success of activity depending on its implementation process and point to the quality of guidance offered by the teacher. Some researchers draw attention to more particular issues related to the tasks used for activities and implementation process. Suzuki and Harnisch (1995), for instance, point to the importance of tasks allowing multiple solution strategies while Baturo et al. (2007) insist on cognitive conflict; Horoks and Robert (2007) are concerned with the use of time and fluency in the transition between stages; Swan (2008) puts an emphasis on tasks that make connections among mathematical concepts and representations; Bozkurt (2018) focuses on the type of directions given to the students to regulate their efforts.

Research studies focusing on the qualities of mathematical activities proposes several features to be considered during design and implementation processes. Bozkurt et al. (2022) propose an evaluation rubric for activity design and implementation and suggested such design principles as use of materials, classroom management, student responsibilities and mathematical outcome. As is the case in this rubric, use of materials is considered to be one of the discriminating components of mathematical activities (see Jorgensen, Dole, & Larkin, 2020; Sullivan et al., 2013; Yazlık, 2018). As Özgen (2017) implies, materials play a decisive role on a variety of activity components ranging from student responsibilities to level of their involvement, emergence of hidden mathematics to the completion of the academic task.

The concept of teaching material is of a detailed use in the studies carried out into mathematical activities. Yeşildere (2018) deals with teaching materials as a phenomenon comprising real world objects together with written, visual, audial, motional, concrete or any kind of digital educational tools. This perspective is compatible with the meanings attached to the notion of teaching materials as conceptualised in the relevant literature on mathematical activity (Doyle, 1983; Henningsen & Stein, 1997; Sullivan et al., 2013). In an evaluation of instructional tools employed in mathematical activities, Özmantar & Bingölbali (2009) argue that these tools are designed or selected to serve particular purposes and consider that they are materials with pedagogical dimensions. Tools play a mediating role in learning and performing particular actions (Hardman, 2019; Wertsch, 1998). As pointed out by the researchers adopting the tradition of Vygotsky, just as tools shape the mind of the users, they are also shaped by the users (Daniels, 2012).

An important framework dealing with the relation between tool and user relationship mentioned here is the theory of instrumental genesis. While examining the interaction between the tool use and learning, instrumental approach focuses on the techniques of individual's tool use and their cognitive development process in an activity (Tabach, 2011). Drijvers & Trouche (2008) highlight two interconnected yet particularly crafted processes called instrumentation and instrumentalization. Instrumentation is conceptualised with regard to effect of instruments on shaping users' thinking. This effect comes into being due to affordances and limitations of tools, which eventually has a formative power on users' working method and knowledge development. As for instrumentalization, it is the process where the instrument is shaped and personalized by the subject (user). Instrumentalization is comprised of four basic stages. The first stage is discovery; the second one is selection; the third one is personalization of the instrument; and the fourth one is the stage of transformation.

As our consideration hitherto suggests, materials used in mathematical activities have profound effects on the design and implementation processes. Jones and Pepin (2016), for instance, point out that teachers' pedagogical design capacities are determined by the tools or materials they decide to use. Swan (2008) goes even further and considers the task itself employed in a mathematical activity as a tool. When viewed from the perspectives of instrumental genesis, tools or materials employed in an activity impose certain limitations on teacher practices as well as provide affordances that enrich teaching in particular dimensions. Despite such deep impacts, we have limited knowledge about how affordances and limitations of tools (including any kind of materials and tasks) prescribed to be employed in an activity are reflected in teachers' selection and preferences of mathematical activities in actual settings. That is, given the important effect of teaching materials on teachers' instructional decisions and practices, empirical knowledge about how teachers approach the materials in their activity selection and which features of the materials are decisive in their preferences is quite limited. Hence this study aims to designate material features determinant in teachers' mathematical activity preferences. This issue is of particular importance, due to the fact that activities selected and implemented by the teachers play an important role in shaping students' perceptions of what mathematics is all about as well as in providing opportunities for students to gain experience of doing mathematics (Stein, 2019). Further to this, gaining insights into the material features prioritised by teachers could allow us to develop a better understanding for the foundations of teachers' pedagogical design capacities (Jones & Pepin, 2016). Besides, when we realise that activities also offer particular learning opportunities to students (Sullivan et al., 2012), this study will make a contribution to our understanding of what material features are taken as references by the teachers in the prescription of these learning opportunities.

# Method

This research was designed as a multiple case study. Case study is considered to be particularly useful to develop theoretical understanding of a real-life phenomenon (Meriam, 2015). Yin (2014) suggests that multiple case studies should be preferred, when possible, over single case studies as multiple case study is a functional method to draw realistic inferences by comparing and contrasting the observations across cases. Multiple cases also provide more analytical benefits than a single case as this method allows corroboration or refutation opportunities to the researchers in terms of research inferences. In this connection, to study the phenomena under investigation to yield a deeper understanding, three secondary school mathematics teachers were chosen as separate cases in this study. In this section, we provide details about the participants, data collection tools and procedures, and data analysis method along with validity and reliability studies.

# **Participants**

In multiple case studies, participants are chosen by a purposeful sampling strategy with a non-random method (Merriam, 2015). The current study was carried out with three secondary school mathematics teachers. Because of privacy reasons, the participants were called as Teacher Merve, Teacher Aslı and Teacher Ümit other than their real names. Teachers were selected through criterion sampling strategy. The selection criteria for the teachers were as follows: (1) working as a mathematics teacher; (2) using activities in teaching; (3) being volunteered; (4) working in the city center (for ease of access). Along with these criteria in the selection of the participants, attention was paid to diversity in terms of educational backgrounds, type of school, professional experience and gender. With these considerations, it was aimed to reach a more detailed understanding by focusing on similar and different evaluations of the participants regarding our focus in this study. Features regarding the participant teachers are summarized in Table 1.

Table 1.

Features of the	1				
Participant	Educational Information	Type of School Worked for	Professional Experience	Gender	
Teacher	Undergraduate Study: Secondary mathematics teacher education	Secondary School	5 years	Female	
Merve	Master's degree: Mathematics education				
	Doctorate: Mathematics education (in progress)				
Teacher Ümit	Undergraduate Study: Secondary mathematics teacher education	Secondary project school (İmam- Hatip)	11 years	Male	
Teacher Aslı	Undergraduate Study: Faculty of Arts and Science (Mathematics dept.)	Private secondary school	1 year	Female	
ASII	Pedagogical Formation Education	school			

### **Data Collection Tools and Procedures**

Data collection from different sources is considered as a basis in case studies (Yin, 2014). In this study, six subsequent semi-structured in-depth interviews were conducted with the participants in order to form a chain of evidence (Yin, 2014) by collecting data from different sources, and the forms (see below) prepared by the participants were collected. To collect data, a series of activities were prepared with different materials but serving to the same mathematical gain. While planning the activities to be used for interviews, three gains from the official curricula document (MEB, 2018) were determined. These gains were chosen because they were considered to have potential to develop mathematical activities with different types of materials. The first gain was "to construct median, bisector and height in a triangle" (#M.8.3.1.1., p.74) from 8th grade. In relation to this gain, we developed activities focusing only on the median in a triangle. The second gain was "to determine the relations between central angles and corresponding arcs and angle measurements" (#M.7.3.3.1, p. 69) from 7th grade. The third gain was "to relate the sum of the length or difference of two sides of triangle with the length of the third side" (#M.8.3.1.2, p.74). For each of these gains a series of activities were developed: 8 activities for the first gain, 10 activities for the second and 6 activities for the third. Activities involved variations in terms of teaching materials and/or prescribed ways of their usage. A pilot study was conducted with two mathematics teachers to check the suitability of the activities to our purpose in this research. Brief descriptions of the activities in relation to target mathematical gains are presented in Table 2.

#### Table 2.

Brief Description of Activities Employed During Data Collection

Gain and Teacher	Activity No	Tools & Materials	Definition of Activity
Aslı: To construct an in a triangle	A1	Geogebra program, computer or smart board	Teacher builds up circles with equal radius first with the help of 12 sign boxes given readily, then builds up bisectors, and finally proves with the help of equal triangles formed.
	A2	Geogebra program, computer or smart board	Teacher builds up bisectors by clicking 3 sign boxes given readily beforehand respectively.
Teacher Aslı: median in	A3	Geogebra program, computer or smart board	Teacher forms a triangle in the interface designed as a blank page and builds up the bisectors of this triangle with the help of a bisector drawing tool given readily beforehand.

# Table 2 continuing

	ontinuing A4	Geogebra	Teacher firstly builds up a triangle and then the bisectors in
		program, computer or smart board	the interface designed as a blank page with the help of menu keys and the instructions on the screen.
	A5	Geogebra program, computer or smart board	Teacher firstly determines the circles in various radius lengths with the help of 9 sign boxes given readily beforehand and then forms the perpendiculars with intersection points. With the help of command button changes the radius lengths of the circles, forms collinear points, and builds up bisectors.
	A6	Protractor, paper, pencil	Students determine angle measures with the help of a protractor divide them to build up suitable bisectors. Students are expected to discover the similarity of building of a bisector in different kind of triangles.
	A7	Compass, paper, pencil	Students draw circles with equal radius with the help of a compass, determine intersection point and build the bisector combining it with the vertex. Students are expected to discover the similarity of building up bisectors in different kind of triangles.
	A8	Paper, pencil, scissors	Students build up bisectors with the help of paper folding in the guidance of teacher. Students are expected to discover the similarity of building up bisectors in different kind of triangles.
	A9	Symmetry mirror, pencil, paper	Students place the symmetry mirror in the guidance of teacher in way to overlap one leg of angle with the display of the other and build up bisectors. Students are expected to discover the similarity of building up bisectors in different kind of triangles.
	A10	Counting sticks	Students make a triangle each with the long counting sticks in the guidance of teacher and build up the bisectors by forming congruent triangles at the corners with the help of short sticks. Students are expected to discover the similarity of building up bisectors in different kind of triangles.
Teacher Merve: to determine the relations between central angles and corresponding arcs and angle measurements	M1	Geogebra program, computer or smart board	There is a ready-made circle marked with center and perimeter angles that see the same arc on this circle. With the help of the checkbox, the ratio between the measures of the angles is determined. It is shown that the ratio between the center and perimeter angle measures does not change.
	M2	Geogebra program, computer or smart board	Through readily given sign boxes, teacher respectively draws a circle, then creates an arc and then central and perimeter angles. It shown that the rate between the measures of central and perimeter angle does not change.
	M3	Geogebra program, Computer, or smart board	Teacher builds up the circle firstly following the instructions given and then the arc, central and perimeter angle corresponding to the arc; then determines the measures of angles. It shown that the rate between the measures of central and perimeter angle does not change.
	M4	Protractor, compass, paper, pencil	Students draw a circle and an arc individually with the help of protractor with the guidance of teacher, and the determine and rate the measure of the central and perimeter angles for the same arc. Students are expected to discover that rate does not change in different lengths of circles and arcs.
	M5	Paper, pencil, scissors	Students draw a circle on the paper with the help of a compass and cut it. Then cut the central and perimeter angles corresponding to the arc and take them out. Folding the central angel into two, they overlap it with the perimeter angle and observe that the rate between them is constant.

Table 2 continuing

	onunung		
	M6 M7	Pizza model, scissors Compass,	Students determine the central and perimeter angles for the same arc out of three circles in the shape of pizza model given them and they cut the perimeter angles of two circles, then they combine them in a way that the sides overlap with the central angle of the third model. They are expected to discover that even if circle and arc change, the rate does not. Students determine the central and perimeter angles for the
sum of the length or difference of two sides of triangle with the length of the third side.	I <b>VI</b> /	paper, colour pencils, scissors	students determine the central and permitter angles for the same arc on the circle they draw and then paint it. They then cut the perimeter angles and combine them to match with the central angle. The relation between the angles is observed. Students are expected to discover that even if circle and arc change, the rate does not.
	M8	Geogebra program, computer or smart board	Readily given circle, arc and the slides formed for the angles are moved in a way to overlap with the perimeter angles for the same arc and central angle. Perimeter angles are moved in a way to overlap with each other and the relation between the measures are explained.
	Ü1	Geogebra program, computer or smart board	Teacher enters the values for lengths as prescribed in the instruction into the input boxes readily given beforehand and forms line segments with the common points. Whether a triangle is formed or not by moving the endpoints is discussed and triangle inequality is discovered.
	Ü2	Geogebra program, computer or smart board	Teacher enters the values for lengths as prescribed in the instruction into the input boxes readily given beforehand and triangle is formed on the screen if the numbers are appropriate if not, "triangle cannot be formed" appears on the screen Students are expected to discover triangle inequality depending on the shaped formed with the change of side lengths.
	Ü3	Geogebra program, computer or smart board	Teacher enters the values for lengths as prescribed in the instruction into the input boxes readily given beforehand and se forming circles taking the endpoints of line segments as the centre and these lengths as radius. With the help of sliding bar radiuses are changed; discussing when the triangles wa formed, the idea of triangle inequality is developed.
sum of the L the length	Ü4	Geometric strips	Students use geometric strips and try to form triangles having the side lengths given in the instructions. With the classroon discussion, students are expected to develop the idea of triangle inequality.
	Ü5	Spaghetti sticks, scissors	Students try to form triangles having the side lengths given in the instructions using spaghetti sticks. With the classroon discussion, students are expected to develop the idea of triangle inequality.
Teacher Umit: to relate the	Ü6	Geogebra program, computer or smart board	Teacher draws circles with suitable centres with the help o menu keys following the instructions in the interface and examines the case of forming triangles with students by focusing on the intersection points of these circles. With the classroom discussion, students are expected to develop the ide of triangle inequality.

The activities as briefly described in Table 2 were prepared as texts containing necessary details with instructions and the materials/tools were provided to the teachers. The data of the study was collected through semi-structured face-to-face interviews with the participant teachers in their schools in a weekly basis. In these interviews, the participants were asked to examine the activities given to them in a week, decide which ones they would use in their classes and to fill a form to explain their preferences and reasons. The interviews were audio-recorded, and the forms filled by the participants were collected. All the participants were informed about that the data obtained will only be used in the process of the research in line with the ethical principles and also it was also declared that no other parties would have

access to the records. The purposes of the interviews carried out with the participants in data collection process and the procedures are given in Table 3.

# Table 3.

Interview	Purpose	Procedure
Week 1	<ul> <li>Informing the participants about the purpose and content of the study</li> <li>Delivering the activities and materials</li> <li>Taking the views of the participants about activity use and intra class practices</li> </ul>	<ul> <li>Participants were informed in general aims for the study and activities were delivered. Geogebra program was set up and the expectations were explained.</li> <li>At the end of the interview, the participants were asked to examine the activities. They were given also a form to indicate the ones that they could use (or not) in their classrooms.</li> <li>Forms given in the previous meeting were</li> </ul>
Week 2	<ul> <li>Taking the views of participants about activities</li> <li>Taking the reasons behind the preferred or dismissed activities</li> </ul>	<ul> <li>collected.</li> <li>An interview was conducted to determine the features taken into consideration while deciding on which activities to use or not in their classes.</li> <li>At the end of the interview, the participants were asked to make an order of priorities for the activities regarding the applicability in actual teaching.</li> </ul>
Week 3	• Determining the features taken into consideration among activities while making a list of priorities	<ul> <li>Forms given in the previous meeting were collected.</li> <li>An interview was conducted to determine the features taken into consideration among activities while making a list of priorities.</li> <li>At the end of the interview, the participants were asked to think about how the materials in activities affected their priority orders and then make any change in their order if they wish.</li> </ul>
Week 4	• Determining how the materials in the activities are reflected on their priority orders.	<ul> <li>Forms given in the previous meeting were collected.</li> <li>An interview was conducted to determine how the materials in the activities are reflected on their priority orders.</li> <li>At the end of the interview, the participants were delivered the materials used in the activities and they were asked to try out the activities by using the materials and then make any change in their order if they wish.</li> </ul>
Week 5	• Allowing participants to experience the activities with the help of researcher	<ul> <li>Forms given in the previous meeting were collected.</li> <li>Each activity was applied with the help of researcher.</li> <li>The goal and scenario of each activity to reach the goal was explained to the participants by the researcher</li> <li>At the end of the interview, the participants were asked to make a final order of priorities regarding the activities by considering their experience in this session taking all the process into consideration and then make any change in their order if they wish</li> </ul>

1000 5 00	intinuing	
	• Determining the changes introduced into the order of	• Forms given in the previous meeting were collected.
	activity preferences	• The evaluations of the participants regarding the
Week 6	• Determining the	changes introduced into in activity orders as a result
	considerations and evaluations of	of experiences were taken and an interview was
	participants for the function of	conducted to find out their reasons for changes if any.
	teaching materials in activities	

# Table 3 Continuing

### **Data Analysis Process**

The data were analyzed by thematic analysis method. The thematic analysis method adopts a phenomenological approach (Creswell, 2016). The most important distinguishing feature of thematic analysis is that it focuses on designation of the notions that could best explain the phenomenon under investigation rather than the frequency of certain codes (Vaismoradi et al., 2013).

In this study, thematic analysis was performed through the stages proposed by Creswell (2016). In this respect, the initial stage was the preparation of the data for analysis. Audio-records of the interviews were transcribed and repeated readings were performed until grasping the data. In order to conduct data analysis, a team of six persons were formed. The team was informed about the purpose of the research and interview scripts were shared. At the second stage, team members were asked to work independently to determine important expressions in relation to research purposes. Sentences or clauses were chosen as unit of analyses and the data were reduced to meaning units through key words. At the third stage, the expression having a significance were examined; meaning units and themes were formed. While determining the meaning units, clauses or expressions referred by the analysis team were considered. At the fourth stage, appropriate quotations were chosen from among the expressions commonly identified by the analysis team as designating the similarities and differences in participants' accounts. These selections were firstly performed by the first author of this paper and then by a second independent coder. The coding differences occurring between the researcher and independent coder were discussed in meetings with the participation of a field expert until reaching a consensus. At the fifth stage, the structural examination of the expressions was carried out. The aim here was to reveal the conditions of teacher decisions, rationale behind choices and the contexts to which teacher accounts referred. Thus, the scope of the previously determined meaning units and the features teachers referred to have been deepened. The final stage of the analysis process was synthesis. Reflections and discussions took place on different combinations that would best explain the similarities and differences between the participants' evaluations. The analysis process was completed by creating passages emphasizing the agreed features as a result of the synthesis.

#### **Reliability and Validity Studies**

As is seen in the details given up to now, the analysis process was carried out by a research team with different background, experience and expertise. The reliability of the research was ensured with the formation of an analysis team, repeated cooperative work cycles, independent coding procedures and transparency. In addition, thick descriptions were provided regarding the research process and the results obtained from the analysis. Detailed descriptions, rearrangement of the raw data according to the themes, and direct quotation without adding any comments were also included in the study as a feature that increases reliability.

Analysing the data by experts and providing the confirmation of the field specialists are important for validity (Özmantar & Batdı, 2020). Supporting the interpretations about the cases determined within the scope of the research with data and the consistency of the findings also increase the internal validity (Silverman, 2016). To ensure internal validity, we had regular meetings with the analysis team, obtained participant confirmation – when felt the need – and coded the data independently as well. The generalization of research results to similar situations is related to external validity (Morse, 2016). In order to achieve this, it is important to share the details of the research. To this direction, detailed descriptions were shared with the reader, and the scope of the study and the procedures followed were explained.

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# **Research Ethics Committee Approval**

In the process of the conduction of the research, all ethical rules were obeyed at the stages of data collection and analysis. In addition, the ethical approval of the research was taken from Gaziantep University, Ethical Committee of Social Sciences and Humanities, dating on 02.04.2021 and permission number of 2021-06-18.

# Findings

The analysis of the data pointed to the five material features determinant in teachers' activity preferences: (1) serving to the mathematical gains, (2) functionality, (3) accessibility, (4) proficiency in use and (5) student familiarity. In this section we delve into each of these features, provide explanations in the light of appropriate quotations from the participant teachers.

### Serving to the Mathematical Gain

Participants put a special emphasis on the issue that materials should serve to the mathematical gains. In their selections and priority orders of the activities, teachers often referred to the instructional potentials of materials employed in the activity. In this respect, teacher Ümit, for instance, made the following comment:

You give a pencil as an example of a line segment, show something like a stick in the class, and it can be an example as its length can be measured but ... you show a pen and the child has a trouble this time since the length of this (the pen) is known. How does it look like a light? Mustn't it go endlessly? I am not sure if the pen here causes a misconception.

Teacher Ümit here made an evaluation of particular materials in terms of communicating a mathematical idea to the students. His judgments considered the extent to which a material could potentially lead to the development of a misconception. The teachers negatively evaluated the situations in which use of materials curtain the real purpose of an activity. In this regard, Teacher Merve expressed that: "I mean students ... started to deal with the compass too much. In this case, they cannot reach out (mathematical) target...I wouldn't use this kind of activities in my instruction" Teacher Merve makes a similar explanation in her evaluations regarding M7 activity in the 3<sup>rd</sup> interview:

I drew the shape and painted ... There is a possibility of mixture of paints ... then which one will be cut in the first place ... They must cut the central angle first, if they start to cut the perimeter angles, there is nothing left... I think that the student will focus on a lot of things that such activities as painting, drawing, cutting will curtain the real purpose of the activity in this way.

Teacher Merve's idea here was that materials and their usage would not shadow the mathematical purpose of the activity. In addition, teachers did not prefer activities which could create a heavy workload due to the materials, inhibiting students from focusing on mathematics. In this sense, teacher Merve's remarks about M6 in the 6<sup>th</sup> interview were as follows:

Children will bring 3 pizzas. They will find separate centres for each one. They will determine the points. So, there is a big workload. I think students' focus here becomes the work itself, I mean, instead of instructional purpose...

As this quotation indicates, whether materials serve to students' mathematical development assumed by the activity could become a matter of concern for teachers. This concern was reflected on teachers' activity choices. Teacher Aslı also shared similar concerns as was clearly expressed in her evaluations in the  $2^{nd}$  and  $4^{th}$  interviews:

The practice and purpose must be integrated with the outcome here. So, what is my purpose and did I reach what I wanted to give at the end ... what we are going to reach while doing the activity, using that material could cause a problem in reaching the real target, this is what I think. I mean they (students) can have some problems in understanding what we are doing, they might not reach to the point we want them to reach.

In her expressions here, Teacher Aslı indicates the mediating role of the material in reaching the desired level of student comprehension with the activity. Teacher Aslı also pointed out that students must not

have problems with the understanding of the works carried out via the material in the content of the activity. While dealing with the serving of material for the purpose, Teacher Ümit also indicated the importance of ease for students' comprehension: "it must be easy to perceive and should not astray them (students) from the outcome, I mean, students should be able to focus the (mathematical) outcome". Teachers expected that materials and their usage would not distract students from focusing on mathematical gains. In this respect, Teacher Ümit made the following comments while explaining why he did not prefer activity Ü6:

This activity looks as if it aims to teach the program of Geogebra, I mean how it is used... it looks like program information. It is as if we teach Geogebra instead of teaching triangle inequality, so I would order it in the last place.

As our considerations hitherto indicate, the materials and their usage, teachers expect, must be related to the mathematical focus of the activity and the target outcome. At the same time, teacher choices were observed to be shaped by whether or not the materials curtain the purpose, shadow the outcome, create irrelevant workload and support students' mathematical growth.

# Functionality

Functionality was another material feature that participants considered while choosing and ordering the activities. Functionality was expressed in terms of affordances and constraints of particular materials employed in an activity. That is, the functionality was conceptualized on the basis of difficulties that material may cause in practice and contributions that materials can make to produce a fluent implementation. Negative aspects such as difficulties experienced in the use of material, its potential to produce errors, to cause unnecessary repetitions for teaching-learning process and to create an extra workload on teachers or students were expressed as features that reduced the functionality of the material. It was observed that the activities considered to employ such kind of materials neither preferred nor prioritised. Teacher Aslı's comments were exemplary in this respect while sharing her evaluations for activity A10:

As a matter of fact, counting sticks are nice but I cannot use it while making bisectors...We cannot find it exactly. It might not be possible to measure with a protractor or with other things since they are always moving and deforming. So, I thought I wouldn't do much progress with the sticks and discarded

Activity A10 deals with the formation of triangles with counting sticks and it also deals with building up a bisector determining the equal distances from these sides with counting sticks as well. Although Teacher Aslı found the sticks "nice", she focused on the possible troubles resulting from deformation of the shapes of the sticks on the table; in other words, sticks would easily move, distorting the constructed triangles. Such concerns negatively affected Aslı's choice of this particular activity.

Functionality effect did not only appear in activities with concrete materials but also in the activities containing technology-supported materials. As an example, teacher Ümit explained the reason why he left activity Ü6 at the bottom of his list as follows:

Ü6 is an activity increasing the challenges. You must start from the very beginning... I mean all the stages must be done flawlessly. If a mistake is made, then everything needs to start from the scratch. This was the reason why I put it to the bottom. It has a lot of challenges...

The activity Ü6 deals with the concept of triangle inequality discovered by using the menu on the dynamic geometry software of Geogebra program through a chain of instructions. Here, Teacher Ümit judged that when students made a mistake in executing prescribed actions, they then had to start the whole process from the beginning. This was considered by Ümit as a formidable constraint of this activity with technology-related materials.

Such positive sides as the fact that realizing the activity with the help of recommended materials to be used in the activity is easy and teaching could be realized without too much effort with the activity thanks to the materials were regarded under the theme of functionality. In this respect, Teacher Ümit explained the reason why he placed  $\ddot{U}4$  activity in the first place in the 2<sup>nd</sup> interview as follows:

It will be effective for a student to see whether a triangle is formed by putting in just two numbers without needing extra information, but just by manipulating the objects on the screen. For this, students need not so much information. I mean it is easy to use for the teacher and student as well. This is the reason I put it in the first place. Ü4 comes to the forefront in terms of practicality and functionality.

Ü4 was an activity formed with the help of the buttons on Geogebra program and based on the drawing of a triangle possible to form with three lengths input. Here Teacher Ümit regarded this activity practical since he thought building up a triangle was easy to perform as prescribed in this activity.

# Accessibility

Accessibility of the material was another feature to which teachers attached importance. Teachers paid attention that the materials involved in activities should be in reach; and if not, they should be at least easily supplied. For that purpose, teachers preferred activities for which the materials were already available in their classrooms or schools. The explanations of Teacher Aslı regarding the accessibility of materials on her preference were as follows: "I prefer what they have in hand rather than asking different things. Like paper, pencils and so, whatever they readily have. I generally prefer such materials to use" Teacher Aslı preferred activities that could be carried out with the materials at her students' disposal and prioritize such activities. Similarly, Teacher Merve also emphasised accessibility of the materials and it was clear from her following statements that she tended to employ real world objects as materials as long as they were readily available: "I sometimes use even a book as a material. I use it while showing the length, the width of it, while making a comparison. I mean anything can be used as a material in the class." In this quotation, Merve stated the importance of availability and ease of access to objects which she could turn into instructional tools instead of bringing materials from outside of the class. On the other hand, teachers' preferences are negatively affected if an activity requires materials only relevant to this particular activity or if the materials need to be supplied from outside of the school either by students or teachers. In this sense, Merve explained the difficulties in supplying materials for particular activities as follows:

I used to bring some materials for certain activities I mean borrowed from the University. I carried lots of them. My students were able to use them, there were sufficient numbers for each of my students. But it was really hard to carry them...bring here and then take back to the University. I am now hesitant because it is really hard, not easy.

Whether students are required to supply materials does also affect teachers' activity choices. Teacher Merve explained that if activity prescriptions required students to bring their own materials, she would be negatively affected. She explained that "accessing pizza model is not so easy to me, I mean if I asked them [students] to bring pizza model to the class, I am sure they wouldn't." Teacher Aslı also shared the same perspective and pointed out particular problems caused by giving students responsibility for the supply of materials: "where we think of a material that is not in the class, I am thinking of this problem. What can we do when there are those who do not bring and hence is not involved?". As could be inferred from teacher statements, they are not favouring activities which ask for materials from the students. Similarly, the lack of materials to be used in the activity at school was regarded negatively. This is clear from Ümit's statements: "if the school is insufficient to provide the necessary materials, this activity is placed as the last choice."

# **Proficiency in Use**

Being proficient in use was another feature that teachers looked for. The fact that teachers have knowledge and experience in the use of materials or that they can use the materials without any problems affects their activity preferences positively. Conversely, teachers' concerns about the teaching practices with the involvement of the materials and the problems that, they think, might be experienced during the implementation stage of the activity are negatively reflected in their decisions and preferences. For instance, Teacher Aslı explained the material effect on activity selection as follows:

I can think of everything in concrete materials, but something is missing about what to do with the technology at some points. You stay behind it at that time, but you see that you can reach the result by making yourself involved in it. On the other hand, I'm not confident ... you search

for it on the smart board or computer, you learn something, but when teaching if you don't use it with the full capacity then what you teach remain shaky.

Teacher Aslı feels self-confident when using concrete materials in activities. However, she had worries and concerns in using technology. Hence, she felt competent while teaching concrete materials; yet she did not feel to have necessary skills to carry out technology related or computer-based activities. Teacher Ümit made an emphasis on his competency while explaining the reasons behind his preferences for the technology-assisted activities: "Would I choose them if I weren't so good in using computer? My interest is that I design web pages and this is my connection with computer. It is an advantage for in using it in my courses." Although Teacher Ümit did not use Geogebra program before, the knowledge and competence he had regarding computer programs came to the forefront. As is seen in these quotations, the activities comprising the materials upon which teachers are competent reflect the preferences of them directly.

#### **Student Familiarity**

Student familiarity with the material was another sought-after feature. It was observed that teachers paid particular attention that students were in good command of material use, had experiences from their previous works, have known the material features relevant to activity. Teachers referred to these characteristics while explaining their reasons in selecting and ranking mathematical activities. For example, teacher Aslı explained the reason why she put A8 activity in the first place in the sixth interviewed as follows:

The reason why I ranked it in the first place is that they had paper in their hand. It is a material known by all and they will not find it strange. As it is a material, they are familiar and it is not a different material, I put it in the first place since I thought they would do it easily.

There is a need to do paper folding work for building bisectors in A8 activity. Teacher Aslı stated clearly that she put this activity in the first place because she thought that her students were used to the paper folding, and because her students could work with this material competently. These ideas were effective on Aslı's activity choices. Teacher predictions about students' difficulties and lack of competencies in using particular materials were observed to have negative effects on teacher preferences of activities. This observation could be traced in Teacher Merve's evaluations about advantages and disadvantages of activities in the fourth meeting:

You must use a compass for example for some activities, but children don't know it, I mean they must learn to use a compass firstly. Then, they must learn to use an protractor... They do not use a ruler, you see. For example, while measuring with ruler, they start counting from one; normally they must start from zero. Let's say you start from one, you can subtract it one at least, but they cannot do it. There are such kind of problems. To me, former teachers did not use it and the student also did not use it at home. I believe the student didn't feel the need to measure a thing.

Teacher Merve thought that her students were not familiar with the use of materials such as compass or ruler and reflected her thoughts in her preference. For that reason, the ideas regarding the level of material familiarity of the students by teacher Merve and for other participants were effective in their activity preferences.

#### **Discussion, Conclusion, and Suggestions**

In this study, our analysis yielded five features of materials sought after by the teachers in their selection and preferences of mathematical activities: (1) serving to the mathematical gains, (2) functionality, (3) accessibility, (4) proficiency in use and (5) student familiarity. Participants' evaluations contributed significantly to our understanding of these features from the practitioners' perspectives. The first determining feature of materials was related to whether prescribed materials served to the mathematical gains. In this respect, participants made evaluations about what the material was, how it was used, what kind of workload it created, how and in what ways it supported students' mathematical growth and understanding. The research has well established that teachers' selection is shaped by a consideration of mathematical gains targeted with activities (Bozkurt, 2012). In our study we observed that teachers approached this aspect at a micro level and made an assessment specific to the materials as well. This suggests to us that mathematical gain of an activity is an issue that demands a holistic consideration. Researchers suggest that components of an activity such as product, student responsibilities and operations (Doyle, 1988) need to be considered while making sound judgments about whether an activity serve to its purpose (Güzel, 2020). Our study revealed that teachers made such judgments by considering the materials prescribed in an activity script as well. Further research on teachers' considerations of other components in relation to mathematical gains would enrich our understanding of teachers' instructional priorities in activity-based teaching.

Functionality of materials was another determining feature for teachers' activity selection. The functionality, generally speaking, has become apparent in teacher evaluations in connection with affordances and constraints of prescribed materials. In this respect, while making judgements about the functionality, the contribution of materials to the conduct of an activity (e.g., ease of use) as well as the actual and potential challenges they may cause (e.g., unnecessary repetitions, extra workload) were taken into consideration. The problems that material use could cause in practice have received research attention directly or indirectly in the related literature and the functionality of materials is found to be an important quality in the success of activity-based teaching (Stein, 2019). For that reason, the evaluations made by the teachers regarding the materials with the perspective of functionality are consistent with the research findings. Considering the mediating role of the materials in any learning process (Hardman, 2019), our study suggests that teachers tended to limit their material choices to the ones judged to be "functional". When viewed from the perspectives of instrumental genesis, it could be argued that the functionality was shaped depending on how materials were personalized by the teachers (Drijvers & Trouche, 2008). As a matter of fact, teachers made evaluations regarding functionality based on the potential contributions and possible limitations imposed by the materials in an activity. From this perspective, it could be stated that teachers' interaction (mental and/or actual) with the materials determine the kind of mathematics at students' disposal, communicate to students what mathematics is all about and how it could be learnt as well as extend or limit the depth of students' understanding (see also Jones & Pepin, 2016). Therefore, it should be pointed out that while functionality is important for a smooth implementation of an activity, judgments in this respect always reflect teachers' personalised interaction with the materials, which, in turn, include important decisions about student's mathematical growth. In the light of these observations, further research will be useful to examine teachers' perceptions of the functionality of materials (such as unit cubes, fraction sets, geometry stripes) available in schools that could be integrated into activity-based teaching.

The third feature that teacher evaluations point to is accessibility. The ease of access to the materials to be used in the activity reflected positively on the preferences of the teachers. Supplying the materials from outside the school negatively affected the activity preferences of the teachers. In addition, it has been seen that materials that could be accessed quickly were preferred in the selection of activities. In relation to this, teachers preferred materials that were naturally found in the classroom in the first stage. If this was not possible, they selected activities using the materials available at the school. If this was not possible, materials that were easy to obtain from outside the school were preferred. This order of preference applies to the materials used by both students and teachers.

It is known that the activities used in teaching play a decisive role in the mathematical development of students (Haggarty & Pepin 2002; Törnroos 2005). Therefore, teacher choices based on the accessibility of the material also determine the learning opportunities offered to students. As stated by Brown and Harris (2009), activities reflect teachers' pedagogical design capacities. Based on observations in our study, it can be argued that the accessibility of the materials has a decisive effect on the pedagogical design capacity of teachers. The issue of accessibility, in and of itself, can also turn into a limiting factor for pedagogical design capacity. Therefore, the possibility of accessing the materials also shapes the mathematical substance of the instruction. Similarly, teachers' pedagogical approaches can be determined based on accessibility. From this point of view, mathematical development of students does not seem to be a sole concern for teachers while selecting activities (see Sullivan et al. 2012); access to material stands out as an important factor shaping this process. Based on our observations and inferences, we believe that it would be informative to conduct further research on the access of different

kinds of materials available in different school types (public-private, city centres-suburbs, primarysecondary etc.) and if, and how, these materials are employed in instructional practices.

Another characteristic that determines teachers' activity preferences is proficiency in using the prescribed materials. Teachers prioritized the materials they had knowledge and experience with. In addition, materials that they thought to be integrated into teaching smoothly and fluently were also preferred. On the other hand, the limitations they felt regarding the use of materials and the assumed material-related difficulties had a negative impact on their activity preferences. As these observations point out, the phenomenon of proficiency refers to a personalized relationship between teachers and materials. As researchers (e.g., Hardman, 2019) emphasize, all tools act as mediators in carrying out certain actions. This also applies to the instructional actions performed using the materials. The actions that teachers perform with materials also shape themselves and their teaching practices. In addition, performing an action cannot be reduced to either tool or user (Werstch, 1998). From this point of view, proficiency also means that the relationship developed based on teachers' previous interactions with the materials is reflected in their preferences. As a matter of fact, studies (eg, Birgin et al., 2020; Kaleli-Yılmaz & Koparan, 2015; Önal, 2014) have shown that teachers who think they are not competent in computer-assisted teaching are reluctant to integrate technology into their instructional practices. As we see it, the notion of proficiency, when viewed from the instrumentalization perspective (Drijvers & Trouche, 2008), corresponds to teachers' quest for the personalization (either imagined or experienced) of a material in terms of the possibilities it offers and the limitations it imposes. On this basis, we argue that quest of proficiency is a visible reflection of the complex relationship that teachers have with the material. We feel an exciting research agenda would be about the limiting effect of the proficiency on teaching practices and the efforts of teachers to expand their proficiency areas.

Student familiarity was another attribute identified in this study. While choosing activities, the teachers gave importance to the students' knowledge of, experience with and competence in using materials. Teachers did not prefer activities when they judged that the materials were not sufficiently familiar to the students who, hence, might experience difficulties. Our observations in this study regarding teachers' privileging the students' familiarity are considered by Özmantar & Bingölbali (2009) in relation to preliminary knowledge of the material. These authors relate successful implementation of activities to the sufficiency of students' prior knowledge about the materials. We recognize that student familiarity could make important contributions to a smooth and fluent implementation of activities. However, an overemphasis on this issue also poses a threat to students' developmental opportunities. This is particularly because materials are tools with a pedagogical dimension (Özmantar & Bingölbali 2009) and hence if familiarity becomes a driving force in the selection of activities, then this situation runs the risk of imposing restrictions on students' opportunities to extend their current understandings which could be supported by the use of different tools (Van de Walle et al., 2016). The fact that Teacher Merve avoided using compass and ruler because her students were not familiar with these tools could illustrate the restrictions we have in mind. We are aware that there is not an easy resolution to avoid such risks. However, we feel it necessary to seek a fine balance between ensuring students' prior knowledge with familiarity and their development opportunities that could be extended with the involvement of new and relevant tools or materials. Further research would be useful to gain empirical insights into if, how and when teachers allow the involvement of new tools or materials into their practices and the conditions under which they are encouraged to make necessary arrangements with this matter.

The findings and discussions shared hitherto showed that teachers' activity preferences had a complex structure and pointed out that instructional decisions were not only shaped on a pedagogical basis and were not only concerned with students' mathematical development. Based on the evaluations of the teachers, it could be concluded that the predictions about the affordances and constraints of the materials were decisive in the activity selection. The affordances of the tools and the limitations they impose are important issues focused by different theoretical perspectives, especially the ones concerned with the use of technological tools employed in instruction. The technological pedagogical content knowledge framework (Mishra & Koehler, 2006) and instrumental genesis theory (Drijvers & Trouche, 2008) are among the examples of these frameworks. According to these frameworks, technological tools have constraints that can negatively affect user performance as much as they provide affordances to support development. In our study, it was seen that teachers took into account the constraints and affordances of

concrete materials as well as technological tools. Teachers' evaluations were shaped based on these two qualities, and in their explanations, positive and negative emphases reflected their insights on these aspects while selecting mathematical activities. However, it is not known to what extent teachers' predictions about the affordances and constraints that guide their choices could be mapped with the realities of actual practices. It would be a productive research agenda to delve into more about teachers' predictions of the constraints and affordances of materials in relations to actual implementation of activities in real settings.

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